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Ion-acoustic solitons and double layers in weakly relativistic multicomponent plasma in presence of electron inertia

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Abstract : Ion-acoustic solitons have been theoretically studied from the Korteweg-deVries (K-dV) and modified Korteweg-deVries (MK-dV) equation in a weakly relativistic plasma consisting of electrons, positive ions and negative ions. The effects of electron inertia and streaming of ions on the formation of solitons in the plasma having (H^+, Cl^-) ions, (H^+, SF_6^-) ions and (Ar^+, O^-) ions have been discussed critically. At the critical situation when nonlinearity in Korteweg-deVries equation vanishes, double layers have been studied from the combined form of K-dV and MK-dV equations. It has been observed that electron-inertia, negative ion concentration and relativistic effects significantly contribute to the formation of double layers.

Keywords : Ion-acoustic soliton, double layers, relativistic plasma

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1. Introduction

Study on the ion-acoustic soliton in cold, collisionless plasma was initiated by Washimi and Taniuti [1] through the derivation of K-dV equation using the reductive perturbation method. Subsequently, experimental investigation was performed by Ikezi *et al* [2] and Ikezi [3] and others from which it was observed that amplitude and width are different from the theoretical values. However, in last two decades, a lot of theoretical and experimental works have been done by various authors incorporating different parameters in the plasma *e.g.*, non-isothermality [4,5], density gradient [6], temperature gradient [7], two temperature electrons [8,9], beam ions [10,11], magnetic field [12–14], negative ions [15,16]. The work of Das and Paul [17] added a new dimension to the study of ion-acoustic soliton as they first introduced the relativistic effect in the plasma.

It is seen that relativistic effect only comes into play in presence of streaming of ions. Following their works Nejoh [18–20], Singh and Dahia [21], Roychowdhury *et al* [22,23], Salahuddin [24], Chakraborty *et al* [25,26], Paul *et al* [27], Mondal *et al* [28], Bera *et al* [29] and others theoretically studied both the ion-acoustic solitons and double layers in a weakly relativistic plasma system, considering various plasma parameters but the effect of electron inertia was considered by Kuehl and Zhang [30], Zhang and Kuehl [31] from which it is observed that the effect of electron inertia is more important than the relativistic effect in the formation on ion-acoustic solitons. They showed that the ion-acoustic soliton solution exists only if the ion drifting velocity is less than the electron thermal velocity. Subsequently, Roychowdhury *et al* [32] investigated double layers in weakly relativistic plasma considering the effect of electron inertia. Later, Kalita *et al* [33] also considered weakly relativistic plasma for the study of ion-acoustic soliton in the presence of electron inertia. They found that both compressive and rarefactive solitons exist at the negligible drifting of electron and ion-streaming motion. However, the effect of electron inertia in a negative ion plasma system has not yet been considered by any previous author. We are, therefore, motivated to study theoretically both ion-acoustic solitons and double layers in a relativistic plasma having negative ions and considering electron inertia. We have noticed that both the profiles of solitary waves and double layers are considerably influenced by the electron inertia as well as relativistic effects.

2. Basic equations

We consider an unmagnetised, collisionless, weakly relativistic plasma having warm positive ions, negative ions and isothermal electrons. In order to study the effect of stream velocity on the ion-acoustic solitary wave we have assumed the ions to have stream velocity but not the electrons to have so. Therefore, for the weakly relativistic plasma, relativistic effects for the positive ions and negative ions will arise and no such effect is to be taken into consideration for the electrons. The temperatures of the ions are much less than the temperature of the electrons and so Landau damping is neglected. Therefore, the governing equations for such a plasma in dimensionless form are :

For positive ions :

$$\frac{\partial \bar{n}_i}{\partial \bar{t}} + \frac{\partial}{\partial \bar{x}}(\bar{n}_i \bar{u}_i) = 0, \tag{1}$$

$$\frac{\partial}{\partial \bar{t}}\bar{u}_{ir} + \bar{u}_i \frac{\partial}{\partial \bar{x}}\bar{u}_{ir} + \frac{\sigma_i}{\bar{n}_i}\frac{\partial \bar{p}_i}{\partial \bar{x}} = -\frac{\partial \bar{\phi}}{\partial \bar{x}}, \tag{2}$$

$$\frac{\partial \bar{p}_i}{\partial \bar{t}} + \bar{u}_i \frac{\partial}{\partial \bar{x}}\bar{p}_i + 3\bar{p}_i \frac{\partial \bar{u}_{ir}}{\partial \bar{x}} = 0. \tag{3}$$

For negative ions :

$$\frac{\partial \bar{n}_j}{\partial \bar{t}} + \frac{\partial}{\partial \bar{x}}(\bar{n}_j \bar{u}_j) = 0, \tag{4}$$

$$\frac{\partial}{\partial \bar{t}}\bar{u}_{jr} + \bar{u}_j \frac{\partial}{\partial \bar{x}}\bar{u}_{jr} + \frac{\sigma_j}{\bar{n}_j}\frac{\partial \bar{p}_j}{\partial \bar{x}} = \frac{Z}{Q}\frac{\partial \bar{\phi}}{\partial \bar{x}}, \tag{5}$$

$$\frac{\partial \bar{p}_j}{\partial \bar{t}} + \bar{u}_j \frac{\partial}{\partial \bar{x}}\bar{p}_j + 3\bar{p}_j \frac{\partial \bar{u}_{jr}}{\partial \bar{x}} = 0. \tag{6}$$

For electrons :

$$\frac{\partial}{\partial \bar{t}}\bar{n}_e + \frac{\partial}{\partial \bar{x}}(\bar{n}_e \bar{u}_e) = 0, \tag{7}$$

$$\frac{\partial}{\partial \bar{t}}\bar{u}_e + \bar{u}_e \frac{\partial}{\partial \bar{x}}\bar{u}_e + \frac{1}{q\bar{n}_e}\frac{\partial \bar{n}_e}{\partial \bar{x}} = \frac{1}{q}\frac{\partial \bar{\phi}}{\partial \bar{x}}. \tag{8}$$

Poisson's equation :

$$\frac{\partial^2 \bar{\phi}}{\partial \bar{x}^2} = \bar{n}_e + \bar{n}_j - \bar{n}_i, \tag{9}$$

where $\bar{u}_{ir} = \bar{u}_i(1 - \bar{u}_i^2/\bar{c}^2)^{-\frac{1}{2}}$

$$\simeq \bar{u}_i\left(1 + \frac{1}{2}\bar{u}_i^2/\bar{c}^2\right),$$

$$\bar{u}_{jr} = \bar{u}_j\left(1 - \bar{u}_j^2/\bar{c}^2\right)^{-\frac{1}{2}}$$

$$\simeq \bar{u}_j\left(1 + \frac{1}{2}\bar{u}_j^2/\bar{c}^2\right),$$

$\sigma_{i,j} = T_{i,j}/T_e$, $\quad Q = m_j/m_i$, $\quad q = m_e/m_i$,

$\bar{x} = x/\lambda_{De}$, $\quad \bar{t} = t/\omega_{pi}^{-1}$,

$$\bar{u}_{e,i,j} = u_{e,i,j}\Big/\left(\frac{K_B T_e}{m_i}\right)^{\frac{1}{2}}, \quad \bar{n}_{e,i,j} = \frac{n_{e,i,j}}{n_0},$$

$$\bar{p}_{i,j} = p_{i,j}/(n_0 K_B T_{i,j}), \quad \bar{\phi} = e\phi/(K_B T_e).$$

$\lambda_{De} = (K_B T_e/4\pi n_0 e^2)^{\frac{1}{2}}$ is the electron Debye length, $\omega_{pi} = (4\pi n_0 e^2/m_i)^{\frac{1}{2}}$ is the ion-plasma frequency, $(K_B T_e/m_i)^{\frac{1}{2}}$ is the ion-sound velocity, n_0 is the unperturbed background electron density, K_B is the Boltzmann constant, $T_{i,j}$ is the temperature of the positive ions and negative ions, T_e is the electron-temperature, m_e, m_i and m_j are the masses of electron, positive ion and negative ion respectively, ϕ is the electrostatic potential, u_i, u_j are the velocity of positive ions and negative ions. The charge neutrality condition is $1 + n_{j0} = n_{i0}$ (for $Z = 1$). Here the particle number densities of the ions have been normalised by the unperturbed number density of electrons. In the subsequent analysis we drop bar on the dimensionless quantities.

3. Ion-acoustic soliton

Now, in order to derive the Korteweg-deVries (K-dV) equation, we use the stretched variables given by Washimi and Taniuti [1] as

$$\xi = \varepsilon^{\frac{1}{2}}(x - V.t), \quad \tau = \varepsilon^{3/2}.t, \tag{10}$$

where V is the phase velocity of the ion-acoustic solitary wave. Moreover, we assume that the physical variables, such as, $n_{e,i,j}$, $u_{e,i,j}$, $p_{i,j}$ and ϕ are perturbed around the equilibrium state as

$$\begin{aligned}
n_e &= 1 + \varepsilon n_{e1} + \varepsilon^2 n_{e2} + \cdots, \\
n_i &= n_{i0} + \varepsilon n_{i1} + \varepsilon^2 n_{i2} + \cdots, \\
n_j &= n_{j0} + \varepsilon n_{j1} + \varepsilon^2 n_{j2} + \cdots, \\
u_e &= \varepsilon u_{e1} + \varepsilon^2 u_{e2} + \cdots, \\
u_i &= u_{i0} + \varepsilon u_{i1} + \varepsilon^2 u_{i2} + \cdots, \\
u_j &= u_{j0} + \varepsilon u_{j1} + \varepsilon^2 u_{j2} + \cdots, \\
p_i &= 1 + \varepsilon p_{i1} + \varepsilon^2 p_{i2} + \cdots, \\
p_j &= 1 + \varepsilon p_{j1} + \varepsilon^2 p_{j2} + \cdots, \\
\phi &= \varepsilon \phi_1 + \varepsilon^2 \phi_2 + \cdots.
\end{aligned} \tag{11}$$

The boundary conditions supplementing the basic equations are

$$\begin{aligned}
&\text{(i) } n_e \to 1, \quad \phi \to 0, \\
&\text{(ii) } n_i \to n_{i0}, \quad n_j \to n_{j0}, \\
&\text{(iii) } u_i \to u_{i0}, \quad u_j \to u_{j0}, \\
&\text{(iv) } p_i \to 1, \quad p_j \to 1,
\end{aligned} \tag{12}$$

as $|x| \to \infty$.

Using (10) and (11) in the basic eqs. (1 9), we get the following equations giving the terms corresponding to the lowest power of ε

$$n_{i1} = n_{i0}\phi_1 / b_i B_i \gamma_{i0},$$

$$n_{j1} = -n_{j0}\phi_1 / b_j B_j \gamma_{j0} Q,$$

$$n_{e1} = \phi_1 n_{e0} / (1 - qV^2),$$

$$u_{i1} = \phi_1 / B_i \gamma_{i0}, \qquad u_{j1} = -\phi_1 / Q B_j \gamma_{j0},$$

$$u_{e1} = \left(\frac{V}{1 - qV^2}\right)\phi_1, \qquad p_{i1} = \frac{n_{i0}}{\sigma_i}\left(\frac{b_i}{B_i} - 1\right)\phi_1,$$

$$p_{j1} = \frac{n_{j0}}{\sigma_j Q}\left(1 - \frac{b_j}{B_j}\right)\phi_1, \tag{13}$$

where $b_i = V - u_{i0}$, $b_j = V - u_{j0}$,

$$B_i = \left(b_i^2 n_{i0} - 3\sigma_i\right) / b_i n_{i0},$$

$$B_j = \left(b_j^2 n_{j0} - 3\sigma_j\right) / b_j n_{j0},$$

$$\gamma_{i0} = 1 + \frac{3}{2} u_{i0}^2 / c^2, \quad \gamma_{j0} = 1 + \frac{3}{2} u_{j0}^2 / c^2$$

From eq. (9), with the substitution of the relations (13) as per requirement, the dispersion relation for the ion-acoustic wave in the plasma is obtained as

$$\frac{n_{i0}}{b_i B_i \gamma_{i0}} + \frac{n_{j0}}{Q b_j B_j \gamma_{j0}} - \frac{n_{e0}}{1 - qV^2}. \tag{14}$$

Then following standard mathematical procedure, we obtain the K-dV equation

$$R_1 \frac{\partial \phi_1}{\partial \tau} + R_2 \phi_1 \frac{\partial \phi_1}{\partial \xi} + \frac{\partial^3 \phi_1}{\partial \xi^3} = 0. \tag{15}$$

where
$$R_1 = \left[\frac{2qVn_{e0}}{\left(1 - qV^2\right)^2} + \left(\frac{1}{b_i A_i \gamma_{i0}} + \frac{n_{i0}}{b_i B_i^2 \gamma_{i0}} + \frac{3\sigma_i}{b_i^3 B_i^2 \gamma_{i0}}\right) + 1/Q\left(\frac{1}{b_j A_j \gamma_{j0}} + \frac{n_{j0}}{b_j B_j^2 \gamma_{j0}} + \frac{3\sigma_j}{b_j^3 B_j^2 \gamma_{j0}}\right)\right],$$

$$R_2 = \left[\left(\frac{2qn_{e0}V^2}{(1 - qV^2)^3} - \frac{n_{e0}}{(1 - qV^2)}\right) + \left(\frac{2}{A_i B_i b_i \gamma_{i0}^2} + \frac{n_{i0}\lambda_i}{B_i^3 b_i \gamma_{i0}^3} - \frac{1}{A_i B_i^2 \gamma_{i0}^2} + \frac{1}{b_i A_i B_i \gamma_{i0}^2} + \frac{3\sigma_i}{B_i^3 b_i^3 \gamma_{i0}^2} + \frac{9\sigma_i}{b_i^3 B_i^3 \gamma_{i0}} + \frac{9u_{i0}\sigma_i}{c^2 B_i^3 b_i^2 \gamma_{i0}^3}\right) - \frac{1}{Q^2}\left(\frac{2}{A_j B_j b_j \gamma_{j0}^2} + \frac{n_{j0}\lambda_j}{B_j^3 b_j \gamma_{j0}^3} - \frac{1}{A_j B_j^2 \gamma_{j0}^2} + \frac{1}{b_j A_j B_j \gamma_{j0}^2} + \frac{3\sigma_j}{B_j^3 b_j^3 \gamma_{j0}^2} + \frac{9\sigma_j}{b_j^3 B_j^3 \gamma_{j0}} + \frac{9u_{j0}\sigma_j}{c^2 B_j^3 b_j^2 \gamma_{j0}^3}\right)\right],$$

$$\frac{\cdots}{n_{j0}} \,|\, B_{i,j}, \tag{16}$$

$$\lambda_{i,j} = 1 + 9u_{i,j0}^2 / 2c^2 - 3Vu_{i,j0} / c^2,$$

$$\lambda_{i,j0} = 1 + 3u_{i,j0}^2 / 2c^2, \quad \sigma_i = \sigma_j = \sigma_0$$

From (15), it is seen that electron-inertia has some contribution on the formation of ion-acoustic soliton in the plasma. It is to be noted that equation (15) becomes identical with equation (6) of Mondal *et al* [28], when the term of the electron-inertia is neglected, *i e*, $q = 0$

From (15), the soliton solution is obtained as

$$\phi_{1(\text{K-dV})} = \phi_{01}\,\text{sech}^2(g\xi - h\tau)/\delta_1 = \phi_{01}\,\text{sech}^2\psi, \tag{17}$$

where $\psi = (g\xi - h\tau)/\delta_1$,

$$\phi_{01} = 3(h/g)(R_1/R_2), \tag{18}$$

and
$$\delta_1 = 2\left[(g^3/h)(1/R_1)\right]^{\frac{1}{2}}. \tag{19}$$

ϕ_{01} and δ_1 are the amplitude and width of soliton respectively.

Relation (17) represents the solitary wave solution of the K-dV equation (15) and the corresponding soliton profiles have been depicted in Figures 1a and 1b The solitary waves

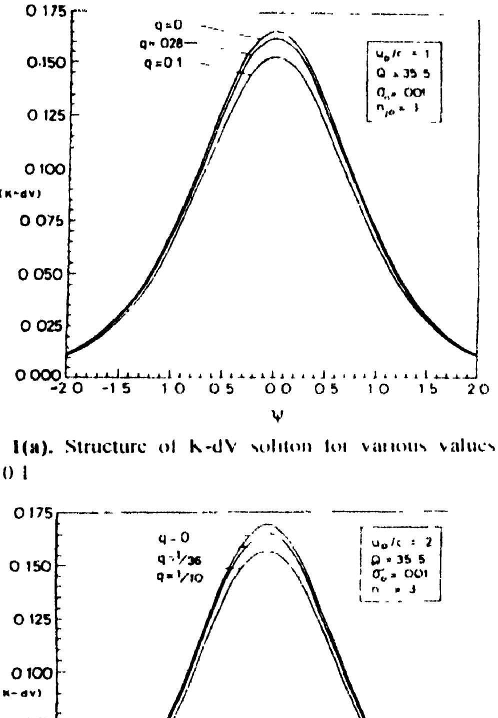

Figure 1(a). Structure of K-dV soliton for various values of q when $u_0/c = 0\ 1$

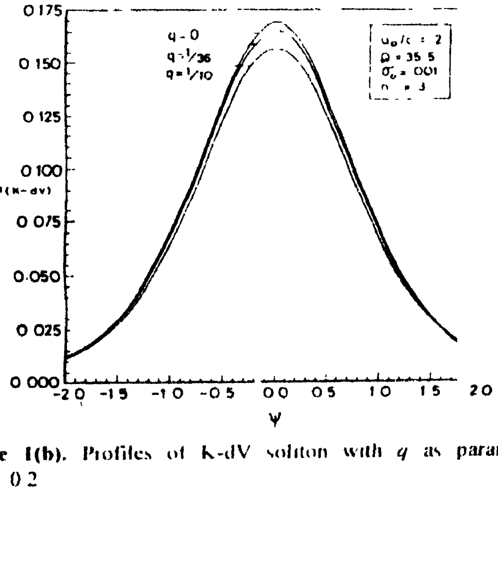

Figure 1(b). Profiles of K-dV soliton with q as parameter when $u_0/c = 0\ 2$

as shown in these figures have been obtained for q = (0.028 and 0.1) for the plasma containing (H^+, Cl^-) ions with negative ion concentration (0.3), ionic temperature (0.001), but for different values of u_0/c (0.1 and 0.2). It has been highlighted in the figures that electron inertia plays a significant role in the formation of soliton. It is noticed that the peaks of the profiles are lowered down with the increase in the value of q.

The variation of the amplitudes of the K-dV soliton represented by the eq. (18) with n_{i0} for fixed values of u_0/c, σ_0 and Q have been shown in the Figures 3(a) and 3(b).

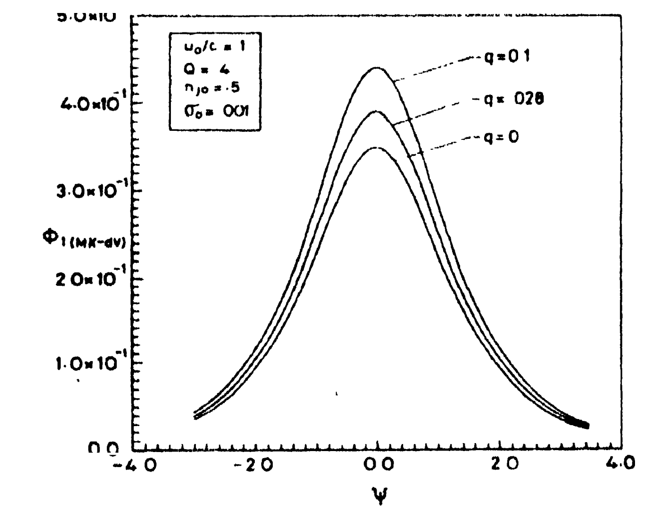

Figure 2(a). Structure of MK-dV soliton for different values of q when u_0/c = 0 1 and n_{i0} = 0 5

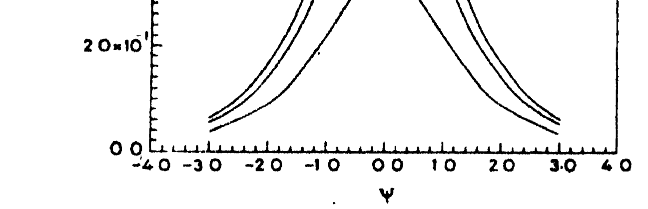

Figure 2(b). Profiles of MK-dV soliton with q as parameter when u_0/c = 0 2 and n_{i0} = 0.9.

A close study of the curves reveals some important informations which are the following :

Firstly, for the values of $n_{i0} > 0.2$, the amplitude increases with the increase of negative ion concentration but it gets reduced as q increases.

Secondly, the rate of increase of amplitude with the variation of negative ion concentration is faster for u_0/c = 0.2 than that for u_0/c = 0.1.

Figures 5(a) and 5(b) show the mode of dependence of widths of the K-dV soliton represented by the eq. (19) on

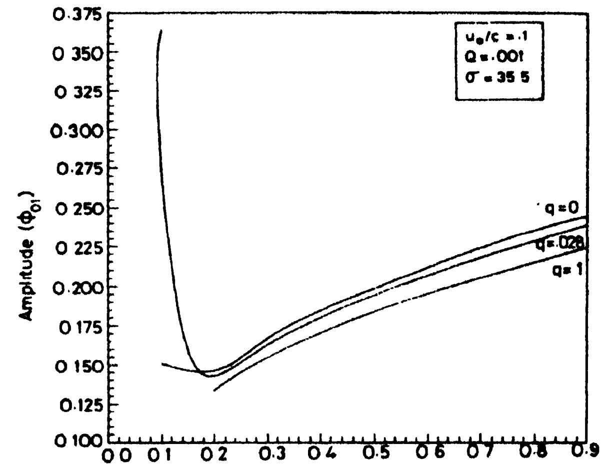

Figure 3(a). Variation of amplitude of K-dV soliton with n_{i0} for various values of q when u_0/c = 0 1

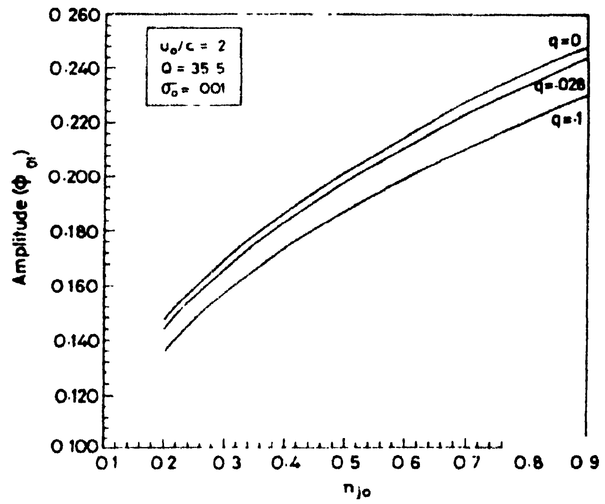

Figure 3(b). Change of amplitude of K-dV soliton with n_{i0} with q as parameter when u_0/c = 0 2

various plasma parameters. It is found that the nature of variations of widths with the plasma parameters are almost similar as compared to the case of amplitude.

4. Modified ion-acoustic soliton and double-layer at critical condition

From (15), we observe that the nonlinear term vanishes where R_2 = 0. Under this situation, ion-acoustic soliton solution cannot be obtained from (15). From the condition (R_2 = 0) of non-existence of ion-acoustic soliton, some critical values of the density of negative ions, stream velocity of the ions and ionic temperature may be obtained. However, solitary wave solution in the plasma near the critical condition may be obtained if we use new scaling variables, *e.g.*,

$$\xi = \varepsilon(x - Vt) \text{ and } \tau = \varepsilon^3 . t. \qquad (20)$$

Using eq. (20) in eqs. (1) to (9) and same perturbation expansion of the dependent variables as in (11) we obtain the equations of the order or ε and find the values of the field variables which are seen to be same as (13).

Now, from the equations obtained by equating the coefficients of ε^2, the field variables come out to be

$$n_{i2} = K_9\phi_1^2,\ n_{l2} = K_7\phi_1^2,\ n_{e2} = (K_9 - K_7)\phi_1^2,$$
$$u_{i2} = K_{10}\phi_1^2,\ u_{l2} = K_{11}\phi_1^2,\ u_{e2} = K_{14}\phi_1^2,$$
$$p_{i2} = K_{12}\phi_1^2,\ p_{l2} = K_{13}\phi_1^2,$$
$$\phi_2 = \frac{B_i\gamma_{i0}}{n_{i0}}(K_9 b_i - K_2)\phi_1^2 \qquad (21)$$

or $$\frac{QB_l\gamma_{l0}}{n_{l0}}(K_3 - K_7 b_l)\phi_1^2$$

and it is also found that $R_2\phi_1 \frac{\partial\phi_1}{\partial\xi} = 0$.

Following the earlier works of the present authors [17,28] we have the MK-dV equation as

$$R_1\frac{\partial\phi_1}{\partial\tau} + R_3\phi_1^2 . \frac{\partial\phi_1}{\partial\xi} + \frac{\partial^3\phi_1}{\partial\xi^3} = 0, \qquad (22)$$

where $$R_3 = \Bigg[\Bigg\{\frac{1}{(1-qV^2)}(2K_1 - 3K_9 + 3K_7) + \frac{V}{(1-qV^2)^2}(qn_{e0}K_{14} + V(K_9 - K_7)q + 3n_{e0}K_{14})$$
$$\frac{n_{e0}V^2}{(1-qV^2)^4}\Bigg\} \; \frac{}{b_i^2B_i\gamma_{i0}}(3K_{10}n_{i0} + 3b_iK_9 - L_1)$$
$$+ \frac{1}{b_l^2B_l\gamma_{l0}}\left(\frac{3K_{11}n_{l0}}{Q} + \frac{3b_lK_7}{Q} + L_2\right)\Bigg].$$

where $$K_1 = \frac{n_{e0}}{2}\left[1 - \frac{3qV^2}{(1-qV^2)^3}\right.$$

$$K_2 = \frac{1}{2}\Bigg[\frac{n_{i0}\lambda_i}{B_i^3\gamma_{i0}^3} + \frac{3n_{i0}}{b_iB_i^2\gamma_{i0}^2} - \frac{n_{i0}}{B_i^3\gamma_{i0}^2} + \frac{3\sigma_i}{b_i^2B_i^3\gamma_{i0}^2}$$
$$- \frac{9\sigma_i}{b_i^2B_i^3\gamma_{i0}} + \frac{9u_{i0}\sigma_i}{c^2b_iB_i^3\gamma_{i0}^3}\Bigg],$$

$$K_3 = \frac{1}{2Q^2}\Bigg[\frac{n_{l0}\lambda_l}{B_l^3\gamma_{l0}^3} + \frac{3n_{l0}}{b_lB_l^2\gamma_{l0}^2} - \frac{n_{l0}}{B_l^3\gamma_{l0}^2} + \frac{3\sigma_l}{b_l^2B_l^3\gamma_{l0}^2}$$
$$+ \frac{9\sigma_l}{b_l^2B_l^3\gamma_{l0}} \; \frac{9u_{l0}\sigma_l}{c^2b_lB_l^3\gamma_{l0}^3}\Bigg],$$

$$K_4 = \frac{n_{i0}}{B_i\gamma_{i0}} - \frac{b_in_{e0}}{1-qV^2},$$

$$K_5 = \frac{n_{i0}n_{l0}}{QB_iB_l\gamma_{i0}\gamma_{l0}} - K_4\left(\frac{n_{l0}}{QB_l\gamma_{l0}} - \frac{b_ln_{e0}}{1-qV^2}\right.$$

$$K_6 = \Bigg[K_4\left(\frac{K_1n_{l0}}{QB_l\gamma_{l0}} + \frac{K_3n_{e0}}{1-qV^2}\right.$$
$$\frac{n_{l0}}{QB_l\gamma_{l0}}\left(\frac{K_1n_{i0}}{B_i\gamma_{i0}} - \frac{K_2n_{e0}}{1-qV^2}\right)\Bigg],$$

$$K_7 = K_6 / K_5,$$

$$K_8 = \left[(K_1 + K_7)\frac{n_{l0}}{B_i\gamma_{i0}} - \frac{K_2n_{e0}}{(1-qV^2)^2}\right], K_9 = K_8 / K_4,$$

$$K_{10} = \frac{b_iK_9}{n_{i0}} \; \frac{1}{b_iB_i^2\gamma_{i0}^2}\Big),$$

$$K_{11} = \Big| \frac{K_7b_l}{n_{l0}} \; \frac{1}{Q^2b_lB_l^2\gamma_{l0}^2}\Big),$$

$$K_{12} = \frac{3\gamma_{i0}K_{10}}{b_i} + \frac{1}{2}\left(\frac{3}{b_i^2B_i^2\gamma_{i0}} + \frac{}{b_i^2B_i^2} + \frac{9u_{i0}}{c^2b_iB_i^2\gamma_{i0}^2}\right)\Big|,$$

$$K_{13} = \frac{3\gamma_{l0}K_{11}}{b_l} \; \frac{1}{2Q^2}\left(\frac{}{b_l^2B_l^2\gamma_{l0}} + \frac{}{b_l^2B_l^2} + \frac{9u_{l0}}{c^2B_l^2b_l\gamma_{l0}^2}\right.$$

$$K_{14} = \left[\frac{V(K_9 - K_7)}{n_{e0}} \; \frac{V}{(1-qV^2)^2}\right],$$

$$L_1 = \Bigg[\frac{b_i^2K_9}{B_i} - \frac{\lambda_in_{i0}}{B_i^3\gamma_{i0}^3} + \frac{3Vn_{i0}b_i}{2c^2B_i^3\gamma_{i0}^3} - \frac{n_{i0}b_iK_{10}}{B_i\gamma_{i0}}$$
$$+ \frac{9b_i^2u_{i0}n_{i0}K_{10}}{c^2B_i\gamma_{i0}} - 3K_9b_i + 2K_2 - \frac{2K_{12}\sigma_i}{B_i\gamma_{i0}}$$
$$- K_{10}n_{i0}\left(\frac{b_i}{B_i} - 1\right) + 6K_{10}\gamma_{i0}n_{i0}\left(\frac{b_i}{B_i} - 1\right.$$
$$\frac{3\sigma_iK_{12}}{B_i} \; \frac{27\sigma_iK_{10}u_{i0}}{c^2B_i\gamma_{i0}} \; \frac{9u_{i0}n_{i0}}{c^2B_i^2\gamma_{i0}^2}$$
$$(b_i / B_i - 1) - \frac{9\sigma_i}{2c^2B_i^3\gamma_{i0}^3}\Bigg],$$

$$L_2 = \Bigg[\frac{3b_lK_7}{Q} - \frac{2K_3}{Q} - \frac{b_l^2K_7}{QB_l} + \frac{\lambda_ln_{l0}}{Q^3B_l^3\gamma_{l0}^3}$$
$$- \frac{3Vb_ln_{l0}}{2c^2Q^3B_l^3\gamma_{l0}^3} + (n_{l0}k_{11}b_l / Q\gamma_{l0}B_l)$$
$$-(9b_l^2u_{l0}n_{l0}k_{11} / c^2QB_l\gamma_{l0}) + (2\sigma_lk_{13} / Q\gamma_{l0}B_l)$$
$$-(k_{11}n_{l0} / Q)(1 - b_l / B_l) - (6k_{11}\gamma_{l0}n_{l0} / Q)$$
$$\times (1 - b_l / B_l) + (3\sigma_lk_{13} / QB_l) + (27k_{11}\sigma_lu_{l0} /$$
$$c^2Q\gamma_{l0}B_l) - (9u_{l0}n_{l0} / c^2Q^2B_l^2\gamma_{l0}^2)$$
$$\times (1 - b_l / B_l) + 9\sigma_l / 2c^2Q^3\gamma_{l0}^3B_l^3\Bigg]. \qquad (23)$$

It is also to be noted that eq. (22) reduces to eq. (9) of Mondal *et al* [28] when electron inertia is neglected, *i.e* $q = 0$.

The MK-dV eq. (22) gives the ion-acoustic soliton solution as :

$$\phi_{1(\text{MK-dV})} = \phi_{02}\text{sech}[(\mu\xi-\lambda\tau)/\delta_2] = \phi_{02}\text{sech}\,\psi, \tag{24}$$

where $\psi = (\mu\xi-\lambda\tau)/\delta_2$,

$$\phi_{02} = [6^{1/2}(\mu/\delta_2)R_3^{-1/2}], \tag{25}$$

$$\delta_2 = [(\mu^3/\lambda)(1/R_1)]^{1/2} \tag{26}$$

μ and λ are the wave number and frequency of the ion-acoustic wave.

In order to study the behaviour of the ion-acoustic wave in the vicinity of the critical situation (when $R_2 \to 0$), we derive a mixed form of the K-dV and the MK-dV equation [34]. The second order momentum eqs. (2), (5) and (8) under the modified scaling variables (20) get transformed into

For positive ions :

$$(-b_i n_{i0}\gamma_{i0}\partial u_{i2}/\partial\xi)+(\sigma_i\partial p_{i2}/\partial\xi)+(n_{i0}\partial\phi_2/\partial\xi) -(b_i\gamma_{i0}n_{i1}\partial u_{i1}/\partial\xi)+(\lambda_i n_{i0}u_{i1}\partial u_{i1}/\partial\xi) +(n_{i1}\partial\phi_1/\partial\xi) = R_2\phi_1\partial\phi_1/\partial\xi. \tag{27}$$

For negative ions :

$$(-b_j n_{j0}\gamma_{j0}\partial u_{j2}/\partial\xi)+(\sigma_j\partial p_{j2}/\partial\xi) -(n_{j0}/Q\partial\phi_2/\partial\xi)-(b_j\gamma_{j0}n_{j1}\partial u_{j1}/\partial\xi) +(\lambda_j n_{j0}u_{j1}\partial u_{j1}/\partial\xi)-(n_{j1}/Q\partial\phi_1/\partial\xi) = R_2\phi_1\partial\phi_1/\partial\xi. \tag{28}$$

For electrons :

$$(-Vq\partial u_{e2}/\partial\xi)+(\partial n_{e2}/\partial\xi)-(\partial\phi_2/\partial\xi) -(Vqn_{e1}\partial u_{e1}/\partial\xi)+(qu_{e1}\partial u_{e1}/\partial\xi) (-n_{e1}\partial\phi_1/\partial\xi) = R_2\phi_1\partial\phi_1/\partial\xi. \tag{29}$$

If we assume that the order of $R_2 \sim O(\varepsilon)$, the order of the right hand side of eqs. (27)–(29) becomes $O(\varepsilon^3)$ and is zero in $O(\varepsilon^2)$. In this situation, the term $R_2\phi_1\partial\phi_1/\partial\xi$ has to be included in the next higher order equation of motion. In these circumstances, eqs. (2), (5) and (8) are reduced to

For positive ions :

$$[(n_{i0}\gamma_{i0}\partial u_{i1}/\partial\tau)-(b_i n_{i0}\gamma_{i0}\partial u_{i3}/\partial\xi)+(\sigma_i\partial p_{i3}/\partial\xi) +(n_{i0}\partial\phi_3/\partial\xi)-(b_i n_{i1}\gamma_{i0}\partial u_{i2}/\partial\xi) +(n_{i0}\gamma_{i0}u_{i1}\partial u_{i2}/\partial\xi)-(b_i\gamma_{i0}n_{i2}\partial u_{i1}/\partial\xi) +(\lambda_i n_{i1}u_{i1}\partial u_{i1}/\partial\xi)-((3V/2c^2)n_{i0}u_{i1}^2\partial u_{i1}/\partial\xi) +(n_{i0}u_{i2}\partial u_{i1}/\partial\xi)-(3b_i/c^2)u_{i0}n_{i0}(\partial/\partial\xi(u_{i1}u_{i2})) +(n_{i2}\partial\phi_1/\partial\xi)+(n_{i1}\partial\phi_2/\partial\xi)]+[(-b_i n_{i0}\gamma_{i0}\partial u_{i2}/\partial\xi) +(\sigma_i\partial p_{i2}/\partial\xi)+(n_{i0}\partial\phi_2/\partial\xi)-(b_i\gamma_{i0}n_{i1}\partial u_{i1}/\partial\xi) +(\lambda_i n_{i0}u_{i1}\partial u_{i1}/\partial\xi)+(n_{i1}\partial\phi_1/\partial\xi)] = 0. \tag{30}$$

For negative ions :

$$[(n_{j0}\gamma_{j0}\partial u_{j1}/\partial\tau)-(b_j n_{j0}\gamma_{j0}\partial u_{j3}/\partial\xi)+(\sigma_j\partial p_{j3}/\partial\xi) -1/Q(n_{j0}\partial\phi_3/\partial\xi)-(b_j n_{j1}\gamma_{j0}\partial u_{j2}/\partial\xi) +(n_{j0}\gamma_{j0}u_{j1}\partial u_{j2}/\partial\xi)-(b_j\gamma_{j0}n_{j2}\partial u_{j1}/\partial\xi) +(\lambda_j n_{j1}u_{j1}\partial u_{j1}/\partial\xi)-((3V/2c^2)n_{j0}u_{j1}^2\partial u_{j1}/\partial\xi) +(n_{j0}u_{j2}\partial u_{j1}/\partial\xi)-(3b_j/c^2)u_{j0}n_{j0}(\partial/\partial\xi(u_{j1}u_{j2})) -1/Q(n_{j2}\partial\phi_1/\partial\xi+n_{j1}\partial\phi_2/\partial\xi)]+[(-b_j n_{j0}\gamma_{j0}\partial u_{j2}/\partial\xi) +(\sigma_j\partial p_{j2}/\partial\xi)-1/Q(n_{j0}\partial\phi_2/\partial\xi)-(b_j\gamma_{j0}n_{j1}\partial u_{j1}/\partial\xi) +(\lambda_j n_{j0}u_{j1}\partial u_{j1}/\partial\xi)-1/Q(n_{j1}\partial\phi_1/\partial\xi)] = 0. \tag{31}$$

For electrons :

$$[(q\partial u_{e1}/\partial\tau)-(Vq\partial u_{e3}/\partial\xi)+(\partial n_{e3}/\partial\xi)-(\partial\phi_3/\partial\xi) -(Vqn_{e1}\partial u_{e2}/\partial\xi)-(Vqn_{e2}\partial u_{e1}/\partial\xi)+(u_{e1}\partial u_{e2}/\partial\xi) +(u_{e2}\partial u_{e1}/\partial\xi)+(n_{e1}u_{e1}\partial u_{e1}/\partial\xi)-(n_{e1}\partial\phi_2/\partial\xi) -(n_{e2}\partial\phi_1/\partial\xi)]+[-(Vq\partial u_{e2}/\partial\xi)+(\partial n_{e2}/\partial\xi) -(\partial\phi_2/\partial\xi)-(Vqn_{e1}\partial u_{e1}/\partial\xi)+(qu_{e1}\partial u_{e1}/\partial\xi) -(n_{e1}\partial\phi_1/\partial\xi)] = 0. \tag{32}$$

Now eliminating the third order terms u_{i3}, u_{j3}, u_{e3}, n_{i3}, ..., ϕ_3 and using (13), (21) and finally normalising τ by R_1, we obtain a mixed MK-dV equation from eqs. (30–32) and other third order equations,

$$\partial\phi_1/\partial\tau + R_2\phi_1\partial\phi_1/\partial\xi + R_3\phi_1^2\partial\phi_1/\partial\xi + \partial^3\phi_1/\partial\xi^3 = 0. \tag{33}$$

This nonlinear equation which involves both quadratic and cubic nonlinearity gives rise to formation of double layer in the vicinity of critical condition that may be characterised by either critical velocity or critical ion density or critical temperature (ion temperature normalised by electron-temperature) given by $R_2 = 0$. Numerical method has to be applied in this case

In order to obtain the double layer solution from (33), we use the transformation $\eta = \xi - \omega\tau$. Eq. (33) is then transformed into

$$\frac{1}{2}\left(\frac{\partial\phi_1}{\partial\eta}\right)^2 + V(\phi_1,\omega) = 0, \tag{34}$$

where V is the Sagdeev potential,

$$V = (1/12)R_3\phi_1^4 + (R_2/6)\phi_1^3 - (\omega/2)\phi_1^2.$$

Double layer solution of eq. (34) is

$$\phi_{1(\text{DL})} = \frac{1}{2}J(R_2)\phi_m(1-\tanh x), \tag{35}$$

where $x = [(-1/24)R_3]^{\frac{1}{2}}\phi_m(\eta)$.

Relation (24) is the solution of the modified K-dV equation (22) and the corresponding solitary wave profiles are depicted in Figures 2(a) and 2(b) for different values of

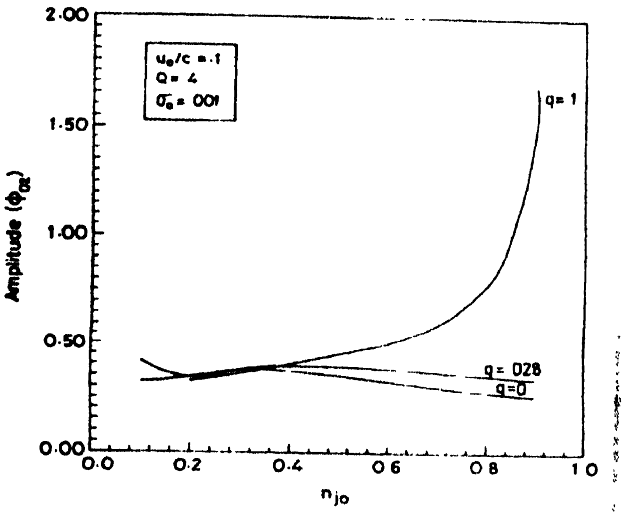

Figure 4(a). Variation of amplitude of MK-dV soliton with n_{j0} for various values of q when u_0/c = 0.1

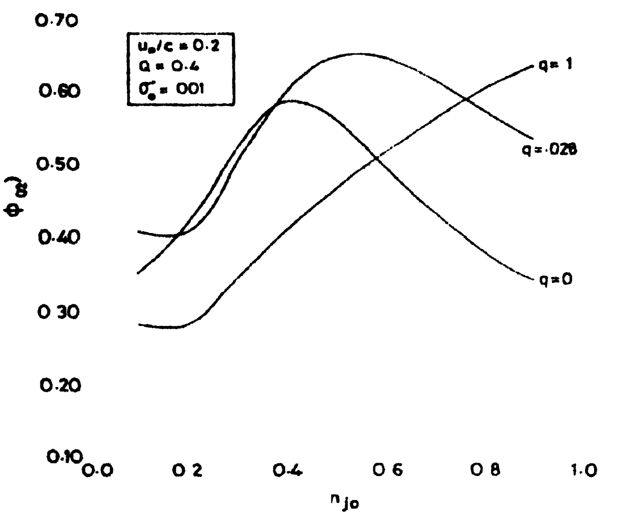

Figure 4(b). Change of amplitude of MK-dV soliton with n_{j0} for different values of q when u_0/c = 0.2.

the plasma parameters. The soliton structures as shown in these figures have been obtained for q = 0, 0.028 and 0.1, in the plasma containing (Ar^+, O) ions, negative ion concentration n_{j0} (= 0.5 and 0.9), ionic temperature (0.001) and different values of u_0/c (0.1 and 0.2). Here, we find that the peaks of the profiles get depressed as q decreases. This is the striking novelty which differentiates the MK-dV soliton profiles from the K-dV soliton profiles. In Figures 4(a) and 4(b) variation of amplitudes of the MK-dV soliton with various parameters have been displayed. The amplitudes are given by the eq. (25). In Figure 4a, the curves obtained for different values of q exhibit two types of dependence of amplitude on n_{j0}. For q = 0 and 0.028 the amplitudes are found to increase though very slightly with n_{j0} for the first time but then to decrease as n_{j0} increases whereas for q = 0.1 sharp increase in width is observed to take place for higher negative ion concentration. In Figure 4b, we find

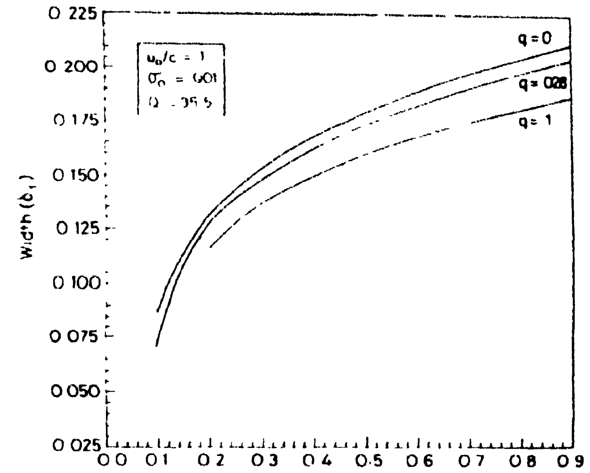

Figure 5(a). Variation of width of K-dV soliton with n_{j0} for various values of q when u_0/c = 0.1

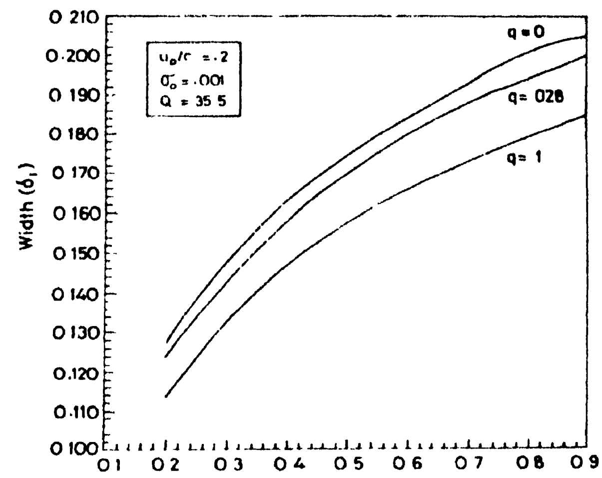

Figure 5(b). Change of width of K-dV soliton with n_{j0} for different values of q when u_0/c = 0.2

amplitudes to change in a similar but prominent way for q 0, 0.028 whereas for q = 0.1, amplitude goes on increasing with the increase in n_{j0}. Figures 6a and 6b show variation of the width of the modified K-dV soliton with different plasma parameters where the widths are represented by the eq (26). For low values of q, widths are seen to decrease first, say for n_{j0} < 0.2 and then to increase as n_{j0} increases but for higher q, widths are found to decrease with increasing n_{j0} though at a slower rate. It is to be noted here that for large Q (say, 127) variation of width with n_{j0} is not appreciable with the exception for q = 0.1. The structures of double layers represented by the eq. (35) have been depicted in Figures 7a and 7b. The profiles are obtained for different

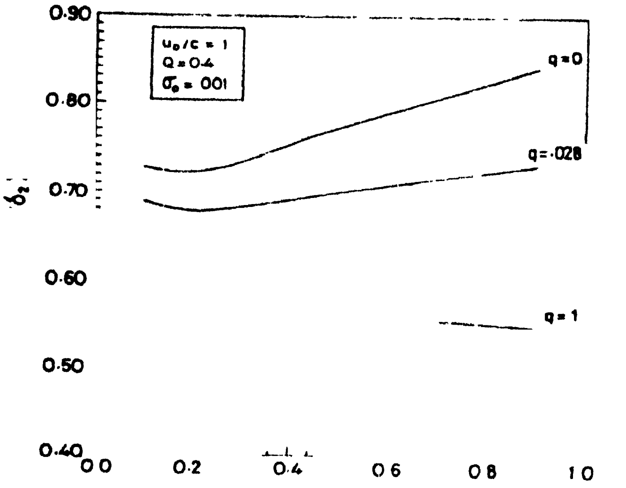

Figure 6(a). Variation of width of MK-dV soliton with n_{j0} for various values of q when Q = 0 4

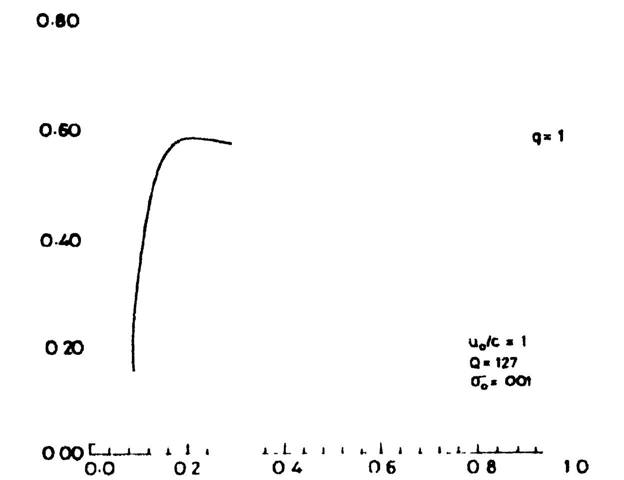

Figure 6(b). Change of width of MK-dV soliton with n_{j0} for different values of q when Q = 127

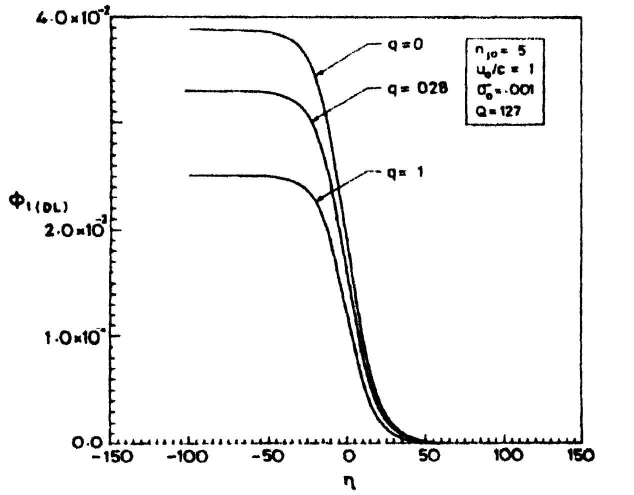

Figure 7(a). Profiles of double-layers with q as parameter when Q = 127

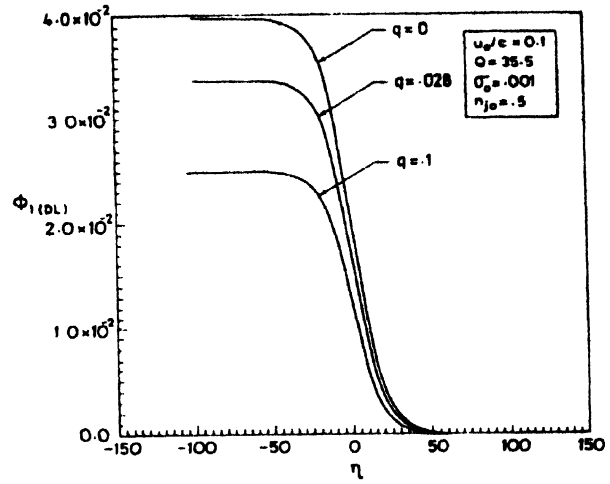

Figure 7(b). Profiles of double-layers for various values of q when Q = 35 5.

values of the plasma parameters. It is evident from the figures that electron inertia has considerable impact on the formation of ion-acoustic double layers.

5. Conclusion

In this paper, we have investigated ion-acoustic solitons and double layers in a weakly relativistic multicomponent plasma consisting of electrons, positive ions and negative ions with streaming motion. The effect of electron inertia has been taken into account for the study of solitons and double layers. In our case, the electrons are warm and isothermal. But, Schamel [4] and other authors showed that electrons may be trapped due to some nonlinear effect and in such cases resonant electrons strongly interact with the wave and Boltzmann distribution for the electron density in an isothermal plasma will not be valid. It is known that experimental values of the amplitude, width and velocity of the solitary wave differ from those predicted theoretically.To remove such discrepancy between the theoretical and the experimental results, higher order contributions of nonlinearity and dispersiveness should be considered [35]. There are numerous experimental results on solitons but due to nonavailability of the same for a relativistic plasma, our theoretical results can not be compared with the experimental data. However, in astrophysical situation, *e.g.*, during solar bursts, electrons and ions attain relativistic velocity and our theoretical results may be applicable there. The charged particles in a plasma are seen to undergo acceleration in space. Formation of double layers (DL) may be one of the causes of particle acceleration in space. So, if double-layers are formed in a weakly relativistic plasma, species of such plasma will be accelerated and may attain ultrarelativistic velocity which may give rise to diverse astrophysical phenomena.

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